Questions

Q1.

(a) Use the binomial expansion, in ascending powers of x, to show that

$$\sqrt{(4-x)} = 2 - \frac{1}{4}x + kx^2 + ...$$

where k is a rational constant to be found.

(4)

A student attempts to substitute $x = 1$ into both sides of this equation to find an approximate value for $\sqrt{3}$.

(b) State, giving a reason, if the expansion is valid for this value of x.

(1)

(Total for question = 5 marks)

Q2.

(a) Use binomial expansions to show that $\sqrt{\frac{1+4x}{1-x}} \approx 1 + \frac{5}{2}x - \frac{5}{8}x^2$

(6)

A student substitutes $x = \frac{1}{2}$ into both sides of the approximation shown in part (a) in an attempt to find an approximation to $\sqrt{6}$

(b) Give a reason why the student **should not** use $x = \frac{1}{2}$

(1)

(c) Substitute $x = \frac{1}{11}$ into

$$\sqrt{\frac{1+4x}{1-x}} = 1 + \frac{5}{2}x - \frac{5}{8}x^2$$

to obtain an approximation to $\sqrt{6}$. Give your answer as a fraction in its simplest form.

(3)

(Total for question = 10 marks)

Q3.

(a) Find the first three terms, in ascending powers of *x*, of the binomial expansion of

$$\frac{1}{\sqrt{4-x}}$$

giving each coefficient in its simplest form.

(4)

The expansion can be used to find an approximation to $\sqrt{2}$
Possible values of *x* that could be substituted into this expansion are:

- $x = -14$ because $\dfrac{1}{\sqrt{4-x}} = \dfrac{1}{\sqrt{18}} = \dfrac{\sqrt{2}}{6}$
- $x = 2$ because $\dfrac{1}{\sqrt{4-x}} = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$
- $x = -\dfrac{1}{2}$ because $\dfrac{1}{\sqrt{4-x}} = \dfrac{1}{\sqrt{\dfrac{9}{2}}} = \dfrac{\sqrt{2}}{3}$

(b) Without evaluating your expansion,

(i) state, giving a reason, which of the three values of *x* should not be used

(1)

(ii) state, giving a reason, which of the three values of *x* would lead to the most accurate approximation to $\sqrt{2}$

(1)

(Total for question = 6 marks)

Q4.

(a) Find the first four terms, in ascending powers of x, of the binomial expansion of

$$(1+8x)^{\frac{1}{2}}$$

giving each term in simplest form.

(3)

(b) Explain how you could use $x = \frac{1}{32}$ in the expansion to find an approximation for $\sqrt{5}$.

There is no need to carry out the calculation.

(2)

(Total for question = 5 marks)

Q5.

$$f(x) = \frac{50x^2 + 38x + 9}{(5x+2)^2(1-2x)} \qquad x \neq -\frac{2}{5} \quad x \neq \frac{1}{2}$$

Given that f(*x*) can be expressed in the form

$$\frac{A}{5x+2} + \frac{B}{(5x+2)^2} + \frac{C}{1-2x}$$

where *A*, *B* and *C* are constants

(a) (i) find the value of *B* and the value of *C*

(ii) show that $A = 0$

(4)

(b) (i) Use binomial expansions to show that, in ascending powers of *x*

$$f(x) = p + qx + rx^2 + ...$$

where *p*, *q* and *r* are simplified fractions to be found.

(ii) Find the range of values of *x* for which this expansion is valid.

(7)

(Total for question = 11 marks)

Mark Scheme

Q1.

Question	Scheme	Marks	AOs
(a)	$\sqrt{(4-x)} = 2\left(1-\frac{1}{4}x\right)^{\frac{1}{2}}$	M1	2.1
	$\left(1-\frac{1}{4}x\right)^{\frac{1}{2}} = 1+\frac{1}{2}\left(-\frac{1}{4}x\right)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(-\frac{1}{4}x\right)^2+\ldots$	M1	1.1b
	$\sqrt{(4-x)} = 2\left(1-\frac{1}{8}x-\frac{1}{128}x^2+..\right)$	A1	1.1b
	$\sqrt{(4-x)} = 2-\frac{1}{4}x-\frac{1}{64}x^2+\ldots$ and $k=-\frac{1}{64}$	A1	1.1b
		(4)	
(b)	The expansion is valid for $\lvert x\rvert < 4$, so $x=1$ can be used	B1	2.4
		(1)	
		(5 marks)	

Notes:

(a)

M1: Takes out a factor of 4 and writes $\sqrt{(4-x)} = 2(1\pm\ldots)^{\frac{1}{2}}$

M1: For an attempt at the binomial expansion with $n=\frac{1}{2}$

Eg. $(1+ax)^{\frac{1}{2}} = 1+\frac{1}{2}(ax)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}(ax)^2+\ldots$

A1: Correct expression inside the bracket $1-\frac{1}{8}x-\frac{1}{128}x^2+$ which may be left unsimplified

A1: $\sqrt{(4-x)} = 2-\frac{1}{4}x-\frac{1}{64}x^2+\ldots$ and $k=-\frac{1}{64}$

(b)

B1: The expansion is valid for $\lvert x\rvert < 4$, so $x=1$ can be used

Q2.

Question	Scheme	Marks	AOs
(a)	$\sqrt{\frac{1+4x}{1-x}}=(1+4x)^{0.5}\times(1-x)^{-0.5}$	B1	3.1a
	$(1+4x)^{0.5}=1+0.5\times(4x)+\frac{0.5\times-0.5}{2}\times(4x)^2$	M1	1.1b
	$(1-x)^{-0.5}=1+(-0.5)(-x)+\frac{(-0.5)\times(-1.5)}{2}(-x)^2$	M1	1.1b
	$(1+4x)^{0.5}=1+2x-2x^2$ and $(1-x)^{-0.5}=1+0.5x+0.375x^2$ oe	A1	1.1b
	$(1+4x)^{0.5}\times(1-x)^{-0.5}=\left(1+2x-2x^2......\right)\times\left(1+\frac{1}{2}x+\frac{3}{8}x^2...\right)$ $=1+\frac{1}{2}x+\frac{3}{8}x^2+2x+x^2-2x^2+...$ $=A+Bx+Cx^2$	dM1	2.1
	$=1+\frac{5}{2}x-\frac{5}{8}x^2......$ *	A1*	1.1b
		(6)	
(b)	Expression is valid $\lvert x\rvert<\frac{1}{4}$ Should not use $x=\frac{1}{2}$ as $\frac{1}{2}>\frac{1}{4}$	B1	2.3
		(1)	
(c)	Substitutes $x=\frac{1}{11}$ into $\sqrt{\frac{1+4x}{1-x}}\approx1+\frac{5}{2}x-\frac{5}{8}x^2$	M1	1.1b
	$\sqrt{\frac{3}{2}}=\frac{1183}{968}$	A1	1.1b
	(so $\sqrt{6}$ is) $\frac{1183}{484}$ or $\frac{2904}{1183}$	A1	2.1
		(3)	
			(10 marks)

(a)

B1: Scored for key step in setting up the process so that it can be attempted using binomial expansions

This could be achieved by $\sqrt{\frac{1+4x}{1-x}}=(1+4x)^{0.5}\times(1-x)^{-0.5}$ See end for other alternatives

It may be implied by later work.

M1: Award for an attempt at the binomial expansion $(1+4x)^{0.5}=1+0.5\times(4x)+\frac{(0.5)\times(-0.5)}{2}\times(4x)^2$

There must be three (or more terms). Allow a missing bracket on the $(4x)^2$ and a sign slip so the correct application may be implied by $1+2x\pm0.5x^2$

M1: Award for an attempt at the binomial expansion $(1-x)^{-0.5}=1+(-0.5)(-x)+\frac{(-0.5)\times(-1.5)}{2}(-x)^2$

There must be three (or more terms). Allow a missing bracket on the $(-x)^2$ and a sign slips so the method may be awarded on $1\pm0.5x\pm0.375x^2$

A1: Both correct and simplified. This may be awarded for a correct final answer if a candidate does all their simplification at the end

dM1: In the main scheme it is for multiplying their two expansions to reach a quadratic. It is for the key step in adding 'six' terms to produce the quadratic expression. Higher power terms may be seen. Condone slips on the multiplication on one term only. It is dependent upon having scored the first B and one of the other two M's

In the alternative it is for multiplying $\left(1+\frac{5}{2}x-\frac{5}{8}x^2\right)(1-x)^{0.5}$ and comparing it to $(1+4x)^{0.5}$

It is for the key step in adding 'six' terms to produce the quadratic expression.

A1*: Completes proof with no errors or omissions. In the alternative there must be some reference to the fact that both sides are equal.

(b)

B1: States that the expansion may not / is not valid when $|x| > \frac{1}{4}$

This may be implied by a statement such as $\frac{1}{2} > \frac{1}{4}$ or stating that the expansion is only valid when $|x| < \frac{1}{4}$

Condone, for this mark a candidate who substitutes $x = \frac{1}{2}$ into the $4x$ **and** states it is not valid as $2 > 1$ oe

Don't award for candidates who state that $\frac{1}{2}$ is too big without any reference to the validity of the expansion.

As a rule you should see some reference to $\frac{1}{4}$ or $4x$

(c)(i)

M1: Substitutes $x = \frac{1}{11}$ into BOTH sides $\sqrt{\frac{1+4x}{1-x}} \approx 1+\frac{5}{2}x-\frac{5}{8}x^2$ and attempts to find at least one side.

As the left hand side is $\frac{\sqrt{6}}{2}$ they may multiply by 2 first which is acceptable

A1: Finds both sides leading to a correct equation/statement $\sqrt{\frac{15}{10}} = \frac{1183}{968}$ oe $\sqrt{6} = 2\times\frac{1183}{968}$

A1: $\sqrt{6} = \frac{1183}{484}$ or $\sqrt{6} = \frac{2904}{1183}$ $\sqrt{6} = 2\times\frac{1183}{968} = \frac{1183}{484}$ would imply all 3 marks

Watch for other equally valid alternatives for 11(a) including

B1: $(1+4x)^{0.5} \approx \left(1+\frac{5}{2}x-\frac{5}{8}x^2\right)(1-x)^{0.5}$ then the M's are for $(1+4x)^{0.5}$ and $(1-x)^{0.5}$

M1: $(1-x)^{0.5} = 1+(0.5)(-x)+\frac{(0.5)\times(-0.5)}{2}(-x)^2$

....

Or

B1: $\sqrt{\frac{1+4x}{1-x}} = \sqrt{1+\frac{5x}{1-x}} = \left(1+5x(1-x)^{-1}\right)^{\frac{1}{2}}$ then the first M1 for one application of binomial and the second would be for both $(1-x)^{-1}$ and $(1-x)^{-2}$

....

Or

B1: $\sqrt{\frac{1+4x}{1-x}} \times \frac{\sqrt{1-x}}{\sqrt{1-x}} = \sqrt{(1+3x-4x^2)} \times (1-x)^{-1} = \left(1+\left(3x-4x^2\right)\right)^{\frac{1}{2}} \times (1-x)^{-1}$

Q3.

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$\frac{1}{\sqrt{4-x}} = (4-x)^{-\frac{1}{2}} = 4^{-\frac{1}{2}}\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}$	M1	This mark is given for rearranging $\frac{1}{\sqrt{4-x}}$ to attempt a binomial expansion
	$\left(1-\frac{x}{4}\right)^{-\frac{1}{2}} =$	M1	This mark is given for an attempt at a binomial expansion
	$1+\left(-\frac{1}{2}\right)\left(-\frac{x}{4}\right)+\frac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{2}\left(-\frac{x}{4}\right)$	A1	This mark is given for a fully correct binomial expansion
	$\frac{1}{\sqrt{4-x}} = \frac{1}{2}+\frac{1}{16}x+\frac{3}{256}x^2$	A1	This mark is given for a fully correct expansion with the first three terms
(b)(i)	$x=-14$, since the expansion is only valid for $\lvert x\rvert < 4$	B1	This mark is given for the correct value chosen with a correct reason
(b)(ii)	$x=-\frac{1}{2}$, since the smaller value will give the more accurate approximation	B1	This mark is given for the correct value chosen with a correct reason

Q4.

Question	Scheme	Marks	AOs
(a)	$(1+8x)^{\frac{1}{2}} = 1+\frac{1}{2}\times 8x+\frac{\frac{1}{2}\times-\frac{1}{2}}{2!}\times(8x)^2+\frac{\frac{1}{2}\times-\frac{1}{2}\times-\frac{3}{2}}{3!}\times(8x)^3$	M1 A1	1.1b 1.1b
	$=1+4x-8x^2+32x^3+....$	A1	1.1b
		(3)	
(b)	Substitutes $x=\frac{1}{32}$ into $(1+8x)^{\frac{1}{2}}$ to give $\frac{\sqrt{5}}{2}$	M1	1.1b
	Explains that $x=\frac{1}{32}$ is substituted into $1+4x-8x^2+32x^3$ and you multiply the result by 2	A1ft	2.4
		(2)	
			(5 marks)
Notes:			

(a)

M1: Attempts the binomial expansion with $n=\frac{1}{2}$ and obtains the correct structure for term 3 **or** term 4.

Award for the correct coefficient with the correct power of x. Do not accept ${}^nC_r$ notation for coefficients.

For example look for term 3 in the form $\frac{\frac{1}{2}\times-\frac{1}{2}}{2!}\times(*x)^2$ or $\frac{\frac{1}{2}\times-\frac{1}{2}\times-\frac{3}{2}}{3!}\times(*x)^3$

A1: Correct (unsimplified) expression. May be implied by correct simplified expression

A1: $1+4x-8x^2+32x^3$

Award if there are extra terms (even if incorrect).

Award if the terms are listed $1,\ 4x,\ -8x^2, 32x^3$

(b)

M1: Score for substituting $x=\frac{1}{32}$ into $(1+8x)^{\frac{1}{2}}$ to obtain $\frac{\sqrt{5}}{2}$ or equivalent such as $\sqrt{\frac{5}{4}}$

Alternatively award for substituting $x=\frac{1}{32}$ into **both sides** and making a connection between the two sides by use of an = or $\approx$.

E.g. $\left(1+\frac{8}{32}\right)^{\frac{1}{2}} = 1+4\times\frac{1}{32}-8\times\left(\frac{1}{32}\right)^2+32\times\left(\frac{1}{32}\right)^3$ following through on their expansion

Also implied by $\frac{\sqrt{5}}{2}=\frac{1145}{1024}$ for a correct expansion

It is not enough to state substitute $x=\frac{1}{32}$ into " the expansion" or just the rhs "$1+4x-8x^2+32x^3$"

A1ft: Requires a full (and correct) **explanation** as to how the expansion can be used to estimate $\sqrt{5}$

E.g. Calculates $1+4\times\frac{1}{32}-8\times\left(\frac{1}{32}\right)^2+32\times\left(\frac{1}{32}\right)^3$ and multiplies by 2.

This can be scored from an incorrect binomial expansion or a binomial expansion with more terms.

The explanation could be mathematical. So $\frac{\sqrt{5}}{2}=\frac{1145}{1024}\rightarrow\sqrt{5}=\frac{1145}{512}$ is acceptable.

SC: For 1 mark, M1,A0 score for a statement such as "substitute $x=\frac{1}{32}$ into both sides of part (a) and make $\sqrt{5}$ the subject"

Q5.

Question	Scheme	Marks	AOs
(a)(i)	$50x^2+38x+9 \equiv A(5x+2)(1-2x)+B(1-2x)+C(5x+2)^2$ $\Rightarrow B = \ldots$ or $C = \ldots$	M1	1.1b
	$B=1$ and $C=2$	A1	1.1b
(a)(ii)	E.g. $x=0$ $x=0 \Rightarrow 9=2A+B+4C$ $\Rightarrow 9=2A+1+8 \Rightarrow A=\ldots$	M1	2.1
	$A=0$*	A1*	1.1b
		(4)	
(b)(i)	$\frac{1}{(5x+2)^2}=(5x+2)^{-2}=2^{-2}\left(1+\frac{5}{2}x\right)^{-2}$ or $(5x+2)^{-2}=2^{-2}+\ldots$	M1	3.1a
	$\left(1+\frac{5}{2}x\right)^{-2}=1-2\left(\frac{5}{2}x\right)+\frac{-2(-2-1)}{2!}\left(\frac{5}{2}x\right)^2+\ldots$	M1	1.1b
	$2^{-2}\left(1+\frac{5}{2}x\right)^{-2}=\frac{1}{4}-\frac{5}{4}x+\frac{75}{16}x^2+\ldots$	A1	1.1b
	$\frac{1}{(1-2x)}=(1-2x)^{-1}=1+2x+\frac{-1(-1-1)}{2!}(2x)^2+.$	M1	1.1b
	$\frac{1}{(5x+2)^2}+\frac{2}{1-2x}=\frac{1}{4}-\frac{5}{4}x+\frac{75}{16}x^2+\ldots+2+4x+8x^2+\ldots$	dM1	2.1
	$=\frac{9}{4}+\frac{11}{4}x+\frac{203}{16}x^2+\ldots$	A1	1.1b
(b)(ii)	$\lvert x\rvert<\frac{2}{5}$	B1	2.2a
		(7)	
		(11 marks)	

Notes

(a)(i)

M1: **Uses a correct identity** and makes progress using an appropriate strategy (e.g. sub $x=\frac{1}{2}$) to find a value for B or C. May be implied by one correct value (cover up rule).

A1: Both values correct

(a)(ii)

M1: Uses an appropriate method to establish an equation connecting A with B and/or C and uses their values of B and/or C to find a suitable equation in A.

Amongst many different methods are:

Compare terms in x^2 $\Rightarrow 50=-10A+25C$ which would be implied by $50=-10A+25\times$"2"

Compare constant terms or substitute $x=0 \Rightarrow 9=2A+B+4C$ implied by $9=2A+1+4\times 2$

A1*: Fully correct proof with no errors.

Note: The second part is a proof so it is important that a suitable proof/show that is seen.

Candidates who write down 3 equations followed by three answers (with no working) will score M1 A1 M0 A0 (b)(i)

M1: Applies the key steps of writing $\frac{1}{(5x+2)^2}$ as $(5x+2)^{-2}$ and takes out a factor of 2^{-2} to form an expression of the form $(5x+2)^{-2} = 2^{-2}(1+{*}x)^{-2}$ where * is not 1 or 5

Alternatively uses direct expansion to obtain $2^{-2} + \ldots$

M1: Correct attempt at the binomial expansion of $(1+{*}x)^{-2}$ up to the term in x^2

Look for $1+(-2){*}x+\frac{(-2)(-3)}{2}{*}x^2$ where * is not 5 or 1.

Condone sign slips and lack of $*^2$ on term 3. ....

Alt Look for correct structure for 2nd and 3rd terms by direct expansion. See below

A1: For a fully correct expansion of $(2+5x)^{-2}$ which may be unsimplified. This may have been combined with their '*B*'

A direct expansion would look like $(2+5x)^{-2} = 2^{-2}+(-2)2^{-3}\times 5x+\frac{(-2)(-3)}{2}2^{-4}\times(5x)^2$

M1: Correct attempt at the binomial expansion of $(1-2x)^{-1}$

Look for $1+(-1){*}x+\frac{(-1)(-2)}{2}{*}x^2$ where * is not 1

dM1: Fully correct strategy that is dependent on the previous **TWO** method marks.

There must be some attempt to use their values of *B* and *C*

A1: Correct expression or correct values for *p*, *q* and *r*.

(b)(ii)

B1: Correct range. Allow also other forms, for example $-\frac{2}{5} < x < \frac{2}{5}$ or $x \in \left(-\frac{2}{5}, \frac{2}{5}\right)$

Do not allow multiple answers here. The correct answer must be chosen if two answers are offered